Derivative Table

1. $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

2. $\frac{d}{dx}(cu) = c\frac{du}{dx}$

3. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

4. $\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + vw\frac{du}{dx} + wu\frac{dv}{dx}$

5. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

6. **(Chain rule)** If y = f(u) is differentiable on u = g(x) and u = g(x) is differentiable on point x, then the composite function y = f(g(x)) is differentiable and

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

7. **(Chain rule)**

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dw}\frac{dw}{dx}$$

8. **(Inverse function)** If y = f(x) has a non-zero derivative at x and the inverse function $x = f^{-1}(y)$ is continuous at corresponding point y, then $x = f^{-1}(y)$ is differentiable and:

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$

9. **(Parametric equation)** For the equation $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$, f(t) and g(t) are differentiable and f'(t) ≠ 0, then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.

10. **(Parametric equation)**

$$\frac{d^2y}{dx^2} = \frac{\frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{d^2x}{dt^2}\frac{dy}{dt}}{\left(\frac{dx}{dt}\right)^3} = \frac{x'y'' - x''y'}{(x')^3}$$

11. $\frac{dc}{dx} = 0$

12. $\frac{d}{dx} x^n = nx^{n-1}$

13. $\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$

14. $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

15. $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$

16. $\frac{d}{dx} \sqrt[n]{x} = \frac{1}{n\sqrt[n]{x^{n-1}}}$

17. $\frac{d}{dx} e^x = e^x$

18. $\frac{d}{dx} a^x = a^x \ln a$

19. $\frac{d}{dx} x^x = x^x (1 + \ln x)$

20. $\frac{d}{dx} \ln x = \frac{1}{x}$

21. $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

22. $\frac{d}{dx} \log x = \frac{1}{x} \log e \approx \frac{0.4343}{x}$

23. $\frac{d}{dx} \sin x = \cos x$

24. $\frac{d}{dx} \cos x = -\sin x$

25. $\frac{d}{dx} \tan x = \sec^2 x$

26. $\frac{d}{dx} \sec x = \sec x \tan x$

27. $\frac{d}{dx} \cot x = -\csc^2 x$

28. $\frac{d}{dx} \csc x = -\csc x \cot x$

29. $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

30. $\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$

31. $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

32. $\frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$

33. $\frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2}$

34. $\frac{d}{dx} \csc^{-1} x = -\frac{1}{x\sqrt{x^2-1}}$

35. $\frac{d}{dx} \sinh x = \cosh x$

36. $\frac{d}{dx} \cosh x = \sinh x$

37. $\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$

38. $\frac{d}{dx} \coth x = -\operatorname{csch}^2 x$

39. $\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$

40. $\frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x$

41. $\frac{d}{dx} \sinh^{-1} x = \frac{d}{dx} \ln\left(x + \sqrt{1+x^2}\right) = \frac{1}{\sqrt{1+x^2}}$

42. $\frac{d}{dx} \cosh^{-1} x = \frac{d}{dx} \ln\left(x + \sqrt{x^2-1}\right) = \pm\frac{1}{\sqrt{x^2-1}}, \ |x| > 1$

43. $\frac{d}{dx} \tanh^{-1} x = \frac{d}{dx}\left(\frac{1}{2} \ln \frac{1+x}{1-x}\right) = \frac{1}{1-x^2}, \quad |x| < 1$

44. $\frac{d}{dx} \coth^{-1} x = \frac{d}{dx}\left(\frac{1}{2} \ln \frac{x+1}{x-1}\right) = -\frac{1}{x^2-1}, \quad |x| > 1$

45. $\frac{d}{dx} \operatorname{sech}^{-1} x = \pm\frac{1}{x\sqrt{1-x^2}}, \ |x| < 1$

46. $\frac{d}{dx} \operatorname{csch}^{-1} x = \pm\frac{1}{x\sqrt{x^2+1}}$

47. $\frac{d}{dx} \ln(\sinh x) = \coth x$, $\frac{d}{dx} \ln(\cosh x) = \tanh x$

Higher Derivatives

1. $\frac{d^n}{dx^n}x^m = m(m-1)....(m-n+1)x^{m-n}$

2. $\frac{d^n}{dx^n}\sqrt{x} = (-1)^{n-1}\frac{1\times3\times5\times....\times(2n-3)}{2^n}x^{-\left(n-\frac{1}{2}\right)}$

3. $\frac{d^n}{dx^n}\frac{1}{x} = (-1)^n\frac{n!}{x^{n+1}}$

4. $\frac{d^n}{dx^n}e^x = e^x$

5. $\frac{d^n}{dx^n}e^{ax+b} = a^n e^{ax+b}$

6. $\frac{d^n}{dx^n}a^x = a^x(\ln a)^n$

7. $\frac{d^n}{dx^n}\ln x = (-1)^{n-1}\frac{(n-1)!}{x^n}$

8. $\frac{d^n}{dx^n}\log_a x = (-1)^{n-1}\frac{(n-1)!}{(\ln a)x^n}$

9. $\frac{d^n}{dx^n}\sin x = \sin\left(x+\frac{n\pi}{2}\right)$

10. $\frac{d^n}{dx^n}\cos x = \cos\left(x+\frac{n\pi}{2}\right)$

11. $\frac{d^n}{dx^n}\sinh x = \begin{cases}\sinh x & ,n \text{ is even}\\ \cosh x & ,n \text{ is odd}\end{cases}$,

$\frac{d^n}{dx^n}\cosh x = \begin{cases}\cosh x & ,n \text{ is even}\\ \sinh x & ,n \text{ is odd}\end{cases}$

12. $\frac{d^n}{dx^n}\sin^2 x = -2^{n-1}\sin\left(2x+\frac{n\pi}{2}\right)$

13. $\frac{d^n}{dx^n}\sin mx = m^n\sin\left(mx+\frac{n\pi}{2}\right)$

14. $\frac{d^n}{dx^n}\cos mx = m^n\cos\left(mx+\frac{n\pi}{2}\right)$

15. $y = \tan^{-1}x,\ \frac{d^n y}{dx^n} = (n-1)!\cos^n y\sin\left(ny+\frac{n\pi}{2}\right)$

16. $y = \cot^{-1}x,\ \frac{d^n y}{dx^n} = (-1)^n(n-1)!\sin^n y\sin ny$

17.

$y = e^{ax}\sin bx,\ \frac{d^n y}{dx^n} = \left(a^2+b^2\right)^{\frac{n}{2}}e^{ax}\sin\left(bx+n\tan^{-1}\frac{b}{a}\right)$

18.

$y = e^{ax}\cos bx,\ \frac{d^n y}{dx^n} = \left(a^2+b^2\right)^{\frac{n}{2}}e^{ax}\cos\left(bx+n\tan^{-1}\frac{b}{a}\right)$

19. (Leibnitz Theorem)

$$(uv)^{(n)} = \sum_{i=0}^{n} C_i^n u^{(n-i)}v^{(i)},$$

where $u^{(0)} = u, v^{(0)} = v,\ u^{(r)} = \frac{d^r u}{dx^r}$